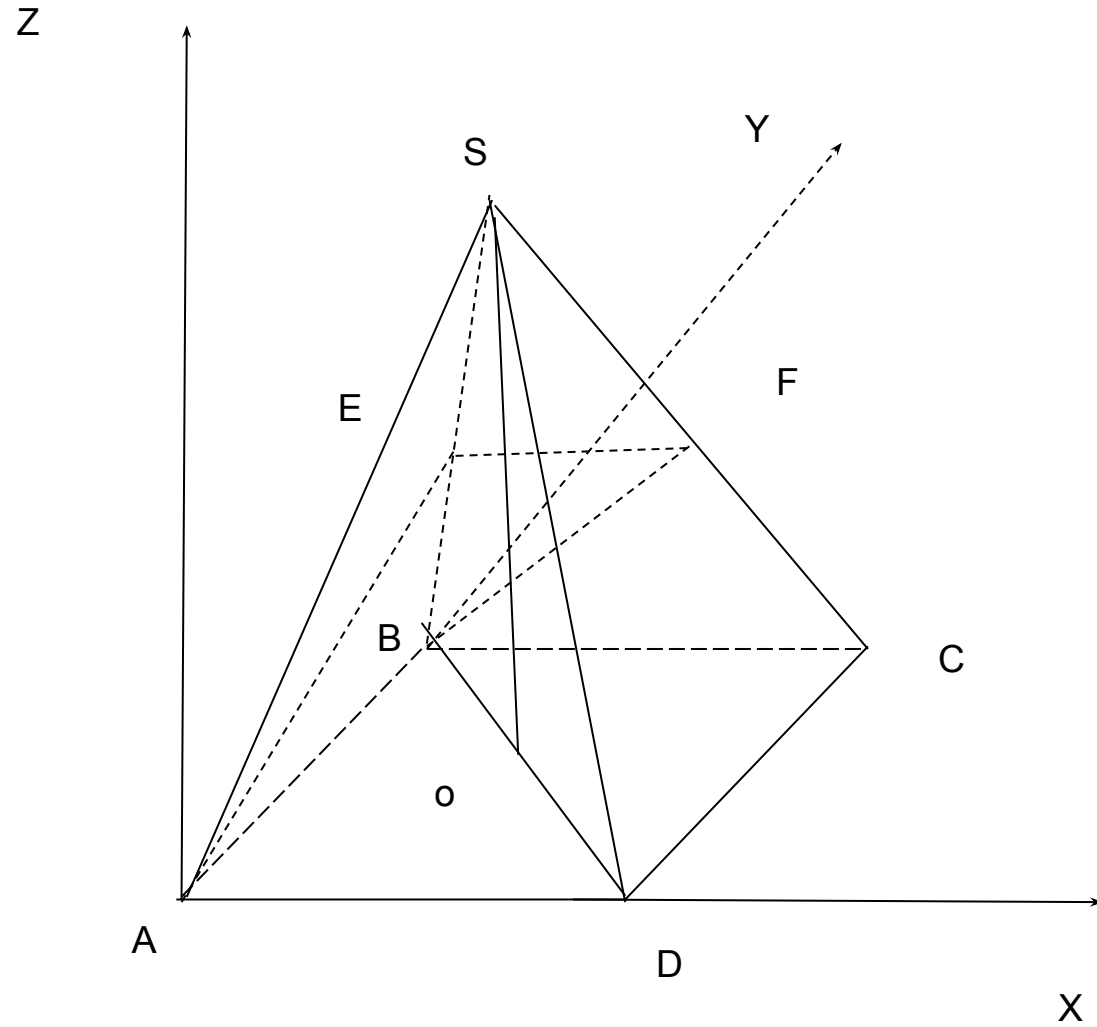
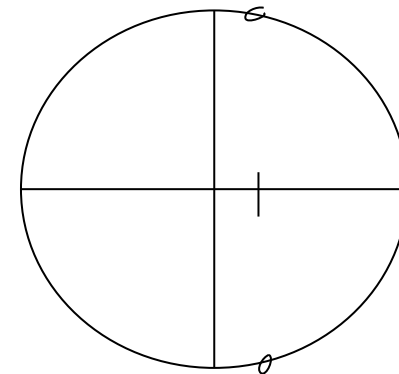


в правильной 4-х угольной пирамиде SABCD, все рёбра которой равны 1, точки E и F - середины рёбер соответственно SB и SC. Найдите косинус угла между прямыми AE и BF



$$\begin{aligned}
 &A(0,0,0) \\
 &B(0,1,0) \\
 &C(1,1,0) \\
 &S\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right) \\
 &SB^2 - BO^2 = OS^2 = 1 - \left(\frac{\sqrt{2}}{2}\right)^2 = 1 - \frac{2}{4} = 1 - \frac{1}{2} = \frac{1}{2} \\
 &OS = \sqrt{\left(\frac{1}{2}\right)} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
 &E(x,y,z) = \left(S\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right) + B(0,1,0) \right) / 2 = \left(\frac{1}{2}, \frac{3}{2}, \frac{\sqrt{2}}{2} \right) / 2 = E\left(\frac{1}{4}, \frac{3}{4}, \frac{\sqrt{2}}{4}\right) \\
 &F(x,y,z) = \left(S\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right) + C(1,1,0) \right) / 2 = \left(\frac{3}{2}, \frac{3}{2}, \frac{\sqrt{2}}{2} \right) / 2 = F\left(\frac{3}{4}, \frac{3}{4}, \frac{\sqrt{2}}{4}\right) \\
 &AE\{E\left(\frac{1}{4}, \frac{3}{4}, \frac{\sqrt{2}}{4}\right) - A(0,0,0)\} = AE\left\{\frac{1}{4}, \frac{3}{4}, \frac{\sqrt{2}}{4}\right\} \\
 &BF\{F\left(\frac{3}{4}, \frac{3}{4}, \frac{\sqrt{2}}{4}\right) - B(0,1,0)\} = BF\left\{\frac{3}{4}, -\frac{1}{4}, \frac{\sqrt{2}}{4}\right\} \\
 &(AE, BF) = |AE| \cdot |BF| \cdot \cos(AE, BF) \\
 &\cos(AE, BF) = (AE, BF) / |AE| \cdot |BF| \\
 &(AE, BF) = \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \left(-\frac{1}{4}\right) + \frac{\sqrt{2}}{4} \cdot \frac{\sqrt{2}}{4} = \frac{3}{16} - \frac{3}{16} + \frac{2}{16} = \frac{1}{8} \\
 &|AE|^2 = \left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{1}{16} + \frac{9}{16} + \frac{2}{16} = \frac{12}{16} = \frac{3}{4} \\
 &|BF|^2 = \left(\frac{3}{4}\right)^2 + \left(-\frac{1}{4}\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2 = \frac{9}{16} + \frac{1}{16} + \frac{2}{16} = \frac{12}{16} = \frac{3}{4} \\
 &|AE| \cdot |BF| = \sqrt{\left(\frac{3}{4}\right)} \cdot \sqrt{\left(\frac{3}{4}\right)} = \frac{3}{4} \\
 &\cos(AE, BF) = \left(\frac{1}{8}\right) / \left(\frac{3}{4}\right) = \frac{4}{24} = \frac{2}{12} = \left|\frac{1}{6}\right| = \frac{1}{6} \\
 &\angle(AE, BF) = \arccos\left(\frac{1}{6}\right)
 \end{aligned}$$

$$\begin{aligned}
 \cos(x) &= \frac{1}{6} \\
 X &= \pm \arccos\left(\frac{1}{6}\right) + 2\pi k
 \end{aligned}$$



OTV: $\arccos(1/6)$